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An Analysis of Jerzy Neyman's Imaginary, Non Existent , Principle of Indifference, Urn Ball Example supposedly taken from J M Keynes's A Treatise on Probability(1921)

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Abstract

Jerzy Neyman analyzed an imaginary, non existent, Urn ball problem that he thought was taken from J M Keynes's *A Treatise on Probability* in his *Lectures and Conferences on Mathematical Statistics and Probability* (1952). Neyman apparently never read the book for himself. He apparently relied on some, other, unknown source to provide him with the problem that he thought came from J M Keynes's *A Treatise on Probability*.

The problem can be analyzed based on an "as if" approach to discover if Neyman realized that Keynes's Principle of Indifference is a substantially different technique from Laplace's concoction that, when applied, will lead to substantially different answers from those obtained by the use of Laplace's Principle of Non –Sufficient Knowledge.

Section 1. Introduction

Jerzy Neyman (JN) spent about one page analyzing a non existent problem for J M Keynes's A Treatise on Probability (1921). How this happened is not clear. However, It shows that JN had no idea about the main differences that existed between Laplace's Principle of Nonsufficient Knowledge(PNSK),based on the use of marginal probabilities and ignorance, and J M Keynes's Principle of Indifference ,(POI),based on knowledge and conditional probabilities. Section Two presents the problem that was supposedly selected from the *A Treatise on Probability* (TP;1921) and makes a few comments speculating on where it could have come from. Section 3 covers Keynes's Principle of Indifference and shows why it is impossible for the problem to be illustrative of Keynes's approach.Section 4 will examine other failed assessments of Keynes's POI.Section 5 concludes the paper.

Section 2.JN's Imaginary problem from J M Keynes's A Treatise on Probability (1921)

How JN came across this problem is unknown to me.However, its errors stand out immediately because Keynes ***neverever***,at anytime in his life ,worked with marginal probabilities. All of Keynes 's probabilities ,which are primarily interval valued probability estimates unless the weight of the evidence, w,equals, approximates or approaches 1,are relative to a body of evidence.Probabiliites can never be generated if there is no knowledge available about them.Keynes's conditional probabilities have to be written,in general , as

P(H/E),

where H is the hypothesis and E is the relevant evidence upon which the probability estimate is based.This means that all of Keynes's probabilities are conditional probabilities.JN 's use of marginal probabilities below, such as P(B),P(Y),and P(W),means that this problem can't possibly be one that was analyzed by J M Keynes in the TP.JN claims the following:

"

Some of the paradoxes solved by Miss Hosiasson are quite amusing. The facility with which one is able to resolve these paradoxes may serve as a test as to whether or not the definition of probability is properly understood. The following paradox is taken from the "Treatise on Probability" by J. M. Keynes (London, 1921, p. 378). Like Dr. Jeffreys, Lord Keynes was also a proponent of the subjective theory of probability.

Consider an urn U of which it is known that it contains exactly n balls. About the color of the balls no information is available. Denote by m the number of black balls in the urn. Because of the complete lack of information as to the color of the balls and since there are $n + 1$ possible hypotheses about the value of m, namely $m = 0, 1, 2, \cdots, n$, the subjective theory of probability ascribes to each of these hypotheses the same probability, namely $1/(n + 1)$. Granting this, it is easy to show that the probability, say $P(B)$ that a ball drawn from the urn will be black is $P(B) = \frac{1}{2}$. This conclusion, by itself, is not questioned. However, Lord Keynes seems to have been puzzled by the circumstance that what applies to black balls should equally apply to white balls and yellow balls. Therefore, if we denote by $P\{W\}$ and $P\{Y\}$ the probabilities that the ball drawn will be white and that it will be yellow, respectively, then $P\{W\} = P\{Y\} = P\{B\} = \frac{1}{2}$.

Further, since the colors white, yellow and black are exclusive, the probability that the ball drawn will be either black, white or yellow would appear to have the absurd value $P\{B + W + Y\} = 1.5$. How come? The reader may wish to try to resolve this "paradox" on his own. If he does not succeed, then he may find it interesting to consult the paper of Miss Hosiasson.

(Neyman,j.1952,p.13)

This example does not appear on p.378 of the TP.It does not appear anywhere in the TP .It does not appear in any of the 30 volumes of the CWJMK It does not appear in either of Keynes's two Fellowship dissertations done at Cambridge University in December,1907 or December,1908.

In many ways ,JN's "analysis" is similar to the error filled analysis of Arne Fisher and Ronald Fisher, presented in book reviews of Keynes's*A Treatise on Probability* (TP;1921) , each of whom was supposed to be a brilliant statistician.

JN does get one thing ,and only one thing ,right.He is correct that Miss Hosiasson did write some interesting papers on probability, although she was never able to figure out that Keynes's probabilities were primarily interval valued probabilities.

Section 3.Keynes 's requirements for applying the POI correctly

Keynes was well aware that Laplace's PNSK approach led to erroneous results. Keynes's POI was set out to correct the many deficiencies that occurred when Laplace's approach was implemented. Keynes would agree wholeheartedly with Reichenbach's judgement , that

"Maybe we have no reason to prefer one face of the die to the other (sic);but then we have no reason to assume that the faces are equally probable ,either. To transform the absence of a reason into a positive reason represents a feat of oratorical art worthy of an attorney of the defense but not permissible in the court of logic."(Reichenbach,1949,p.14).

Laplace's principle is based on equal, balanced, amounts of ignorance. Keynes's principle is based on equal amounts of balanced positive evidence. Laplace's approach used marginal probabilities. Keynes's approach requires conditional probabilities. Laplace's principle has no implicit or explicit concept of the weight of the evidence, w, whereas Keynes's principle makes it explicit. Finally, all of Keynes's probabilities must be based on evidence whereas Laplace's probabilities can be based on ignorance.

Unfortunately, Reichenbach badly confused Laplace's principle with Keynes's principle in his article in much the same way as Jerzy Neyman did. The POI requires the presence of positive, symmetrical evidence that is relevant.(See Reichenbach,1949,pp.14-17). Reichenbach also makes the same mistake as J N when he claims that Keynes is a subjectivist.(Reichenbach,1929,p.202.)

Keynes would argue that positive, symmetrical evidence could be obtained by carefully examining the six different faces of the die. This would lead to knowledge. That this evidence is relevant is easily shown by the following two options. Option one involves gambling on a die (Dice) where you have not been allowed to examine the die (Dice) before playing .Option two would be being allowed to examine the die(dice) before gambling with it .The overwhelming ,vast number of players will select option two. This evidence is relevant.

Keynes's first requirement was that the weight of the relevant evidence, w, had to equal, approximate or approach 1.This is Keynes's requirement for calculating numerical probabilities. If the w<1 ,then interval valued estimates will be the only alternative . Second, all of the relevant evidence has to be symmetrically balanced. Third, all of the probabilities must be conditional probabilities. Finally, the outcomes can't be broken down into smaller parts. They must be indivisible .This requires discrete outcomes like tossing a coin, tossing a die or dice, playing various types of card games, drawing from finite urns, playing with bean bags etc.

JN 's problem above states that

"Consider an urn U of which it is known that it contains exactly n balls. About the color of the balls no information is available. Denote by m the number of black balls in the urn. Because of then complete lack of information as to the color of the balls….the subjective theory of probability ascribes to each of these hypotheses the same probability, namely 1/(n+1)…." (Neyman,J.,1952,p.13).

This means that w,the weight of the evidence ,is 0.For Keynes,but not for Laplace,it is impossible to obtain probable knowledge from ignorance.JN is really confused here because,contrary to his claim above, J M Keynes was ***not*** "…a proponent of the subjective theory of probability"(Neyman,J.,1952,p.13).

If the weight of the evidence is zero, then there is no probability. "We simply do not know" (Keynes,1937,p.213).

JN's view of Keynes, based on a problem that does not exist in the TP,is illustrative of a gross ignorance of Keynes's use of conditional probability. JN claims that

"…

> Keynes seems to have been puzzled by the circumstance that what applies to black balls should equally apply to white balls and yellow balls. Therefore, if we denote by $P\{W\}$ and $P\{Y\}$ the probabilities that the ball drawn will be white and that it will be yellow, respectively, then $P\{W\} = P\{Y\} = P\{B\} = ½$.

"(Neyman,j.1952,p.13)

Of course, if Keynes had been asked to comment on this problem,it would be that the only way that one can possibly proceed is that the probability of drawing a black ball must be conditioned on the knowledge of the existence of the colors of other balls in the urn.An example would be that the probability (of drawing a black ball in the urn,***given that the only known colors are black and white***) is ½.Another example would be that the probability (of drawing a black ball in the urn,***given that the only known colors are black,yellow and white***) is 1/3.

JN's claim that Keynes was perplexed by the result that P(W) =P(Y)= P(B)=1/2 ,so that the sum of the probabilities is 1.5,means that he has no idea about what Keynes was talking about in the TP during his lifetime.J N 's analysis is basically consistent with the conclusion that Laplace and Keynes agreed with each other on the issue of the application of the PNSK and /or POI,which is absurd ,given the heavy criticism levelled by Keynes against Lapace.

J N was certainly one of the top 10 statisticians in the twentieth century.However, his "analysis" of Keynes's "problem" makes him look stupid and foolish.

Section 4. Other erroneous assessments of Keynes's POI

The belief that Keynes rejected his own POI is a common error committed by Post Keynesians. For example, consider the following erroneous logic of Gillies.

It is based on a severe confusion of Laplace's approach , which is based on ignorance, with Keynes's approach ,which is based on positive ,symmetrical knowledge. Gillies reaches his conclusion by taking out of context a partial quote ,from the CWJMK Volume 8 version of the TP ,on p.94, where Keynes is discussing other authors approaches to the use of Laplace's approach ,who were connected to the German historical school. He then badly misinterprets the quote and confuses Laplace's version ,based on ignorance ,with Keynes's version based on knowledge:

“ Before discussing inter subjective probability in this context, however, I shall present a further piece of evidence for the argument that Keynes did abandon the logical interpretation of probability in *The General Theory* .As we saw earlier, Keynes’s version of the logical interpretation of probability makes use of what he called the Principle of Indifference. Admittedly, Keynes does give a full discussion of the paradoxes to which this principle leads, though he is not very successful in resolving these paradoxes. Yet in *A Treatise on Probability* he still regards the Principle of Indifference as essential for probability theory. As the following remarks about it show:

“On the grounds both of its own intuitive plausibility and of that of some of the conclusions for which it is necessary, we are inevitably led towards this principle as a necessary basis for judgements of probability .In *some* sense, judgments of probability do seem to be based on equally balanced degrees of ignorance.”(CWVIII:94)

By contrast, in *The General Theory* Keynes wrote:

“Nor can we rationalize our behavior by arguing that to a man in a state of ignorance in either direction are equally probable, so that there remains a mean actuarial expectation based on equi-probabilities. For it can easily be shown that the assumption of arithmetically equal probabilities based on a state of ignorance leads to absurdities.”(CWVII:152)

This amounts to a complete repudiation of the Principle of Indifference, and it is interesting to note that Keynes may here be echoing Ramsey, who wrote:

“To be able to turn the Principle of Indifference out of formal logic is a great advantage for it is fairly clearly “To be able to turn the Principle of Indifference out of formal logic is a great advantage;for it is fairly clearly impossible to lay down purely logical conditions for its validity,as is attempted by Mr. Keynes.”(Ramsey,1978:91) , as is attempted by Mr. Keynes.”(Ramsey,1978:91)

(Gillies,2003,p.122).

Ramsey is simply confused here .He is trying to evaluate deductively a tool that is used inductively. Ramsey’s claim that Keynes attempted “…to lay down purely logical conditions for its validity” is an obviously false claim that demonstrates Ramsey’s obsession with his belief that there was no ,and could never be, any such thing as inductive logic. Ramsey’s claims have been rejected by all cognitive scientists and cognitive psychologist since the mid 1970’s

Supposedly,

"All this establishes that Keynes did abandon his logical interpretation of probability in light of Ramsey's criticisms." (Gillies,2003,p.122)

Given that Boolean , interval valued probability is the work horse of Keynes's logical system, what is established here is that Gillies simply doesn't know what he is talking about. The POI represents a small portion of Keynes's approach. It's use requires that the weight of the positive evidence,w, have a value of 1.It is very clear that Gillies, like Ronald Fisher, Ramsey ,de Finetti and Bateman, has absolutely no idea about what is involved with the concept of interval valued probability ,which is crucial for Keynes's logical theory of probability. Hehas also conflated Laplace's approach ,based on ignorance ,with Keynes's approach,which is based on positive symmetrical evidence.

Section 5.Conculsion

It should be clear that neither Jerzy Neyman nor Hans Reichenbach actually read the *A Treatise on Probability*.

I believe that their understanding of Keynes's book came from (a) reading book reviews about the TP or (b) reading one or two chapters, usually from the front part of the book. This is the Frank Ramsey approach to studying the TP. He only read chapters 3 and 4 of the TP.The relative lack of influence of Keynes's work ,therefore , results from the ignorance of the potential readers of the TP. Anyone reading Neyman's or Reichenbach's assessment of Keynes's POI most likely wouldcome to the mistaken conclusion that Keynes's book was not worth reading. Nothing could be further from the truth

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