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On J M Keynes's Rejection of the Moscow School of Probability's Limiting Frequency Approach to Probability and Kolmogorov's Axiom of Additivity (Countable Additivity): Non –Additivity was the fundamental, basic axiom upon which all of the Economics of Keynes was Built

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Abstract

J M Keynes was an acknowledged, world renown, and internationally recognized expert in probability and statistics in the 1930's based on his A Treatise on Probability (1921). . Keynes had been selected by statistics journals to serve as a referee during the 1930's. It is, therefore, no surprise that he was selected as the referee by the League of Nations to review Jan Tinbergen's work on business cycles that used an econometrics approach based on The Law of Large Numbers, the Central Limit Theorem, and the Gaussian (Normal) Distribution .The fundamental axiom used by Tinbergen was additivity .

Kolmogorov and the Moscow School of Probability's main innovation was to go from the axiom of additivity to the axiom of countable additivity. However, Keynes rejected additivity except in the special case that the weight of the evidence, w, which measured the relative completeness of the evidence ,defined on the closed unit interval [0,1],equaled 1 , approached 1,or approximated 1. Keynes also accepted goodness of fit tests, such as the Lexis –Q test, and exploratory data analysis as evidence that could be used to support using a particular probability distribution.

Keynes also rejected countable additivity for the same reasons he rejected additivity, in general. Keynes's theory of probability and decision making is based on the interval valued approach of G. Boole, who also rejected additivity except as a limiting case. Thus, non –additivity and non-linearity, or imperfect and incomplete information, used as a shorthand description for economists, are the fundamental axioms on which the economics of Keynes, and Adam Smith before him, are based.

Ergodicity is based on a measure function, which is based on the Kolmogorov and the Moscow School of Probability's Axiom of Countable Additivity. Therefore, it is obvious that Keynes would reject ergodicity (and non ergodicity) as general results unless the researcher provided extensive exploratory data analysis and goodness of fit tests supporting their claims. Paul Davidson and the

Post Keynesian School have never supplied a shred of evidence to support their claims. The only group of researchers to do this are the econophysics followers of Benoit Mandelbrot and Nicholas Nassim Taleb, who were the first to supply empirical evidence to support their analysis.

Therefore, it is easy to completely reject the claims of Paul Davidson, made numerous times since 1983 that Keynes was unaware of the Kolmogorov-Moscow School of probability's main result, which was to move from the axiom of additivity to the axiom of countable additivity, and then on to the concept of ergodicity, based on the definition of a measurable function.

Keynes's main axiom was non additivity, which leads directly to nonlinearity. Keynes's conclusions regarding liquidity preference, his rejection of the conclusion of gross substitutability, and his rejection of the meaningfulness of the ergodic-non ergodic distinction in stochastic process theory, are all based on non additivity and non-linearity. All classical and neoclassical results are special cases based on additivity and /or countable additivity that lead to the linearity of the normal distribution. There are no missing axioms from Keynes's work

Paul Davidson has thus erred continuously over the last 33 years in confusing axioms with conclusions and claiming, without a shred of evidence, historical or otherwise, that Keynes was completely ignorant of the Kolmogorov-Moscow School of probability in the 1930's .Keynes would never have been selected as the reviewer of Tinbergen's work by the League of Nations if that was thought to be the case.

The fundamental axioms of neoclassical economics are additivity and linearity. From these axioms, and no other axioms, one can derive the conclusions of the neutrality of money, gross substitutability, and ergodicity. One could use the assumption of perfect and complete information for all decision makers as a shorthand description of the neoclassical position. All neoclassical schools base their analysis on additivity and linearity .Therefore, Neoclassical economics is a special, limiting case of Keynes's more general theory based on the axioms of non-additivity and non-linearity.

Section 1.) Introduction-Keynes's axiom of Non -additivity

Consider the following assessment made by Keynes in the A Treatise on Probability in chapter 26 concerning the use of mathematics in the social sciences and liberal arts (Keynes's terms these moral sciences) :

"The hope, which sustained many investigators in the course of the

Nineteenth century, of gradually bringing the moral sciences under

the sway of mathematical reasoning, steadily recedes—if we mean,

as they meant, by mathematics the introduction of **precise** numerical

methods. **The old assumptions, that all quantity is numerical and**

that all quantitative characteristics are additive, can be no longer

sustained. Mathematical reasoning now appears as an aid in its

symbolic rather than in its numerical character."[Keynes, 1921,A Treatise on Probability (TP), p.318; bold face and underline added by the author]

Additivity is an assumption that holds in many of the areas of study in the physical, biological, and engineering sciences, but not in the social sciences and liberal arts. Therefore ,the axiom of additivity and the claim built on it, that all quantity is numerical, must be replace with the axiom of non – additivity. This immediately led to the use of interval estimates by Boole and Keynes. However,the reader should note that it was Adam Smith, and not Boole or Keynes, who first recognized this in his discussions in chapters 10 and 11 of Part I of The Wealth of Nations in 1776 in his critique of Richard Cantillon's probabilistic risk based assessments of occupational choice.

There can be no axioms of ergodicity or non ergodicity because both ergodicity or non ergodicity are built on the axiom of additivity. The axiom used by the neoclassical schools of economics follows Jeremy Bentham's emphasis on additivity and quantification. Thus, Keynes's reference to Euclidean, non-Euclidean, and the axiom of parallels (Keynes, GT; 1936, p.16) is actually a contrast between additivity and non-additivity. Keynes's axiom of non additivity can be contrasted with the neoclassical axiom of additivity. All of the neoclassical results, such as neutrality of money and gross substitutes, follow from the assumptions of additivity and linearity.

Keynes's logical theory of probability is built on the foundation erected by George Boole in the Laws of Thought (1854).Boole established that many applied problems that had to be dealt with by decision makers were incomplete and indeterminate. Incomplete information, data, and knowledge of relevant factors was missing. The standard, purely mathematical, theory of probability, upon which statistics is based, asserts that the individual is in possession of all the relevant data before he must make a decision concerning which option to choose from a set of options. The standard mathematical theory of probability assumes additivity and complementarity .Additivity is the crucial axiom. Given a complete set of data, the decision maker can choose the best option from the set of all alternative options. The sum of all the probabilities of the different alternatives is additive since the probabilities must sum to 1. .Boole and Keynes argued that in many cases the decision maker has an incomplete information or data set so that relevant information or data is not available or is missing at the time the actual decision must be made .This led Smith, Boole, and Keynes to develop the concept of interval valued probability.

Keynes introduced interval valued probability in his discussions in chapter 3 of the TP. The concept of interval valued probability in then developed and applied in chapters 15,16,17,20, 22, and 26 of the TP by Keynes. Interval –valued probabilities are non additive and non linear, since the inequality constraints used can involve powers greater than one. See Brady, 1993;Brady,2002;Brady,2004A; Brady 2004B, and Brady and Arthmar,2012 for a technical discussion of Keynes's application of his non additive, nonlinear approach to decision making. See Brady,2012, for a general overview of Keynes's non additive approach.

Section 2.) Kolmogorov's Axiom's

Consider the following summary:

Kolmogorov's axiomatization in his *Foundations of the Theory of Probability* (1933) uses the following set theoretic approach aiming at measure theoretic constructs. Define Ω to be a non-empty set. A *field* (or *algebra*) on Ω is a set **F** of subsets of Ω that has Ω as a member and that is closed under complementation (with respect to Ω) and union. Let *P* be a function from **F** to the real numbers obeying the following axioms:

1. Non-negativity- $P(A) \geq 0$, for all $A \in \mathbf{F}$.
2. Normalization- $P(\Omega) = 1$.
3. Finite additivity- $P(A \cup B) = P(A) + P(B)$ for all $A, B \in \mathbf{F}$ such that $A \cap B = \emptyset$.

Define P to be a *probability function* and (Ω, **F**, *P*) to be a *probability space*.

Now replace Axiom 3. Above with 3.′ Below, requiring it to be closed under complementation and *countable* union. This is defined to be a *sigma field* (or *sigma algebra*) on Ω.

3′. Countable additivity- If $A_1, A_2, A_3 \ldots$ is a countably infinite sequence of (pairwise) disjoint sets, each of which is an element of **F**, then

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n)$$

In other words, Kolmogorov's improvement over previous axiom systems of probability was to generalize from finite additivity to countable additivity.

That is all there is to the Kolmogorov axioms of the Moscow School of Probability. They are practically useless in the most areas of economics, business, finance, social science, liberal arts, and everyday practical decision making. Their realm of application is confined to the Life, Biological, and Physical sciences, like physics, chemistry, biology, and engineering.

Section 3.) Ergodicity (Non Ergodicity) is built on Kolmogorov's application of measure theory to probability and statistics.

Consider the following definition of a measure:

Let *X* be a set and Σ an σ-algebra over *X*. Define a function μ from Σ to the real numbers. It is called a measure if it satisfies the following three properties:

- Non-negativity: For all *E* in Σ: $\mu(E) \geq 0$.
- Null empty set: $\mu(\emptyset) = 0$.
- Countable additivity (or σ-additivity): For all countable collections $\{E_i\}_{i \in \mathbb{N}}$ of pairwise disjoint sets in Σ:

$$\mu\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mu(E_k)$$

.

The pair (*X*, Σ) is called a measurable space. The members of Σ are called measurable sets. If (X, $\sum_{x)}$ and (Y, $\sum_{y}$) are two measurable spaces, then a function f: X→Y is called measurable if for every *Y*-measurable set B ϵ $\sum_{y}$, the inverse image is *X*-measurable – i.e.: $f^{(-1)}$ (B) ϵ $\sum_{y}$ A triple (*X*, Σ, μ) is called a measure space. A probability measure is a measure with total measure one – i.e. μ (*X*) = 1. A probability space is a measure space with a probability measure.

Ergodicity is then constructed and analyzed based on the measure theory results discussed in this section.

Section 4.) Examining the claims made by P. Davidson over the last 33 years about J M Keynes and the Moscow School of Probability

Consider the following claims of P. Davidson below. The reader should note that there are at least 12 other nearly identical articles that will not be dealt with here where Davidson makes the same false claims. All of these claims amount to the argument that Keynes had no idea about what the Moscow School of Probability was up to. Supposedly, Keynes was ignorant of Kolmogorov's new axiomatic foundation for the limiting –relative frequency interpretation of probability, as well as being ignorant of Kolmogorov's measure theoretic extension to probability. This represented a new twist because Kolmogorov added the concept of countable additivity.

Upon this foundation, Kolmogorov built his erotic theory, which is, in fact, a minor advance over previous analysis based on the Law of Large Numbers:

First,

We have Claim # 1:

Keynes laid great stress on the distinction between uncertain and probable events, especially as in relation to decisions involving the accumulation of wealth and the possession of liquidity. In his 1937 defense of *The General Theory*, Keynes wrote:

> By "uncertain" knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty. . . . The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention. . . . About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know! (1973*b*, pp. 114-15)

Keynes' separation of uncertain from probable events was developed in his earlier work *A Treatise on Probability* (1973*a*): a volume in which he strongly attacked the logical basis of the LaPlacian school (the forerunners of modern Bayesian analysis). Unfortunately, in his day, Keynes did not have access to the meticulous work of the Moscow School of Probability which developed in exacting detail the now standard theory of stochastic processes. In retrospect, therefore, we can only seek to reinterpret Keynes' fine intuition of the distinction between uncertain and probable events in terms of such processes.

(Davidson, 1982-83, p.188)

Next is Claim #2:

"Keynes never used the term "ergodic" since ergodic theory was first developed in 1935 by the Moscow School of Probability and it did not become well known in the West until after the Second World War and Keynes was dead. Nevertheless Keynes's main criticism of Tinbergen's econometric "method" [Keynes, 1939a, p. 308] was that the economic data "is not homogeneous over time". Non-homogenous data over time means that economic time series are non-stationary, and non-stationary is a sufficient (but not a necessary condition) for non ergodic circumstances. Consequently, Keynes, with his emphasis on uncertainty had, in these comments on Tinbergen, specifically rejected what would later be called the ergodic axiom …." (Davidson, n.d., p.18)

Next, we have Claim #3:

"Keynes never used the term "ergodic" since ergodic theory was first developed in 1935 by the Moscow School of Probability and it did not become well known in the West until after the Second World War and Keynes was dead. Nevertheless Keynes's main criticism of Tinbergen's econometric "method" [Keynes, 1939a, p. 308] was that the economic data "is not homogeneous over a period of time". This means that economic time series are non-stationary, and non-stationary is a sufficient (but not a necessary condition) for a non ergodic process. Consequently, Keynes, with his emphasis on uncertainty had, in these comments on Tinbergen, specifically rejected what would later be called the ergodic axiom."(Davidson, 2005, p.12)

Next, we can have Claim #4:

> The terminology of the ergodic axiom was explicitly developed by the Moscow mathematical school of probability in 1935, and did not become popular in Western Europe and the United States until well after the Second World War and Keynes had died. Keynes never used this ergodic terminology in his emphasis on the importance of uncertainty and the demand for liquidity in his 1936 book or any other writings. Nevertheless, Keynes noted that 'It would be foolish, in forming our expectations, to attach great weight to matters which are very uncertain. . . . By "very uncertain" I do not mean the same thing as "very improbable".[7] Moreover, as Keynes noted in his criticism of Tinbergen's econometric methodology, no reliable information existed today for providing a reliable forecast of future economic outcomes because economic data is not homogeneous over time.[8] The homogeneity of economic data over time is a necessary condition for an ergodic system.
>
> (Davidson, Paul, 2011, p.38)

Next, we have claim #5:

"Keynes's concept of uncertainty about the economic future requires the economic system to be generated by a non ergodic stochastic process. At the time of his writing *The General Theory*, Keynes did not know of the ergodic stochastic theory that was being developed by the Moscow School of Probability in the 1930s. Nevertheless in his criticism of Tinbergen's [econometric] method, Keynes [1939] wrote that Tinbergen's method is not valid for any economic forecasting because economic data "are not homogeneous" over time. Non homogeneity is a sufficient condition for non ergodicity.

Taleb's Black Swan concept attempts to explain market crashes as an event lying in the far off tail of an ergodic probability distribution. It should be noted that Knight's vision of uncertainty and Taleb's Black Swan concept are both based on the ergodic presumption for the economy. Taleb's Black Swan is an already predetermined outcome but the Black Swan event is so far out in the tail of the ergodic probability distribution that its occurrence is so rare that it is never likely to be observed–

except in the long run when we will all be dead. Similarly Knight's applied his uncertainty concept to an event that is "in a high degree unique" and hence so far out in the distribution as to be observed perhaps only once in several lifetimes." (Davidson, P., 2012, p.62).

Next, we have Claim #6:

"Although in his discussion of uncertainty Keynes did not know or use the dichotomy between an ergodic and nonergodic stochastic system in his criticism of Tinbergen's methodology Keynes notes that economic time series cannot be stationary because "the economic environment is not homogeneous over a period of time". Nonstationarity is a sufficient but not a necessary condition for a nonergodic stochastic process. Accordingly, Keynes was implicitly arguing that economic processes over time occur in a nonergodic economic environment."(Davidson, 2009, p.332)

And, finally, the following claim:

"Keynes never used the term "ergodic" since ergodic stochastic theory was first developed in 1935 by the Moscow School of Probability and it did not become well known in the West until after the Second World War and Keynes was dead. Nevertheless Keynes's main criticism of Tinbergen's econometric "method" (Keynes, 1939a, p. 308) was that the economic data "is not homogeneous over time". Non-homogenous data over time means that economic time series are non-stationary, and non-stationary is a sufficient (but not a necessary condition) for a non ergodic stochastic process. Consequently, Keynes, with his emphasis on uncertainty had, in these comments on Tinbergen, specifically rejected what would later be called the ergodic axiom…" (Davidson, P...2008, p.562).

There are many errors of commission and omission in these quotations from the sampled works of Davidson in this area. It will take several pages to analyze all of them. I ask the reader to bear with me as the demonstration commences.

Davidson's Claim one is completely unsupported by any source material in any of the statistical literature of the 1930's.The Moscow School of Probability's so-called "meticulous " result ,produced in "exacting detail" ,is to replace the axiom of additivity ,assumed by all previous limiting –relative frequency interpretations of probability, from john Venn to Richard Von Mises, with countable additivity. This is then followed by defining a measure space. After this is done, then, and only then, can an analysis of ergodicity begin?

A second claim of Davidson's is that Keynes's separation of uncertainty, defined as an inverse function of the weight of the evidence on p.148 of the General Theory (GT, 1936) by Keynes and linked directly by Keynes on p.148 (p.240) of the GT to Keynes's discussion of weight in chapters 6 (and 26) of the A Treatise on Probability (TP; 1921), from probability was only a "fine intuition "of Keynes's. Since the weight of the evidence, w, must either equal , approximate or approach one before additivity becomes possible, it can be concluded that Davidson simply does not know or understand what Keynes meant by the term "weight". He does not understand the concept of the weight of the evidence. Such a concept is fundamental to all logical approaches to probability. Davidson's claim makes little or no sense because it is impossible to reinterpret Keynes as a frequency interpretation of probability advocate, which is exactly what Davidson is doing. The frequency interpretation has no weight of the evidence criterion. One can, of course, as Keynes did

in chapter 6 of the TP, attempt to provide a criterion in which the weight of the evidence will play a role in frequency theories.

Davidson learned his probability and statistics when he was an undergraduate and graduate student in a Biology department before he switched to the study economics. It is the relative frequency-limiting frequency approach to probability that Davidson accepts as the general case. This simply means that Davidson is, and has been for 60 years, an advocate of an approach to probability that Keynes considered applicable only as a special case. It is only in the case where the only relevant evidence is frequency data that Keynes's approach and the frequency approach coincide. Note that Keynes would always accept frequency statements as part of the relevant data.

Let us now consider the claim two in the second quotation above:

"Keynes's main criticism of Tinbergen's econometric "method" [Keynes, 1939a, p. 308] was that the economic data "is not homogeneous over time". Non-homogenous data over time means that economic time series are non-stationary, and non-stationary is a sufficient (but not a necessary condition) for non ergodic circumstances."

Keynes's major objection to Tinbergen's method was that he had assumed normality without supplying empirical evidence in the form of either Keynes's preferred goodness of fit test, the Lexis – Q test, or any other goodness of fit test or exploratory data analysis. Bunch maps simply do not do the job. Davidson's claim that

".. Non stationary is a sufficient (but not a necessary condition) for non ergodic circumstances." has been known to be false since the 1973 paper of Madsen and Isaacson.

Let us now move on to claim Three. Consider the following claim by Davidson:

"Non-stationary is a sufficient (but not a necessary condition) for a non ergodic process." This is simply a repeat of the previous error. It is false. Davidson has repeatedly made this basic error many times over the last 33 years.

Let us now move on to claim Four. Consider the following full quotation from p.148 of the GT supposedly presented by Davidson:

"It would be foolish, in forming our expectations, to attach great weight to matters which are very uncertain [1]. It is reasonable, therefore, to be guided to a considerable degree by the facts about which we feel somewhat confident, even though they may be less decisively relevant to the issue than other facts about which our knowledge is vague and scanty. For this reason the facts of the existing situation enter, in a sense disproportionately, into the formation of our long-term expectations; our usual practice being to take the existing situation and to project it into the future, modified only to the extent that we have more or less definite reasons for expecting a change."(Keynes, 1936, p.148)

The very important footnote that is missing from Davidson's exposition above is

1. **By "very uncertain' I do not mean the same thing as "very improbable". Cf. my Treatise on Probability, chap. 6, on "The Weight of Arguments".** (Keynes,GT,p.148,fn.1)

Let us look at Davidson's treatment:

"It would be foolish, in forming our expectations, to attach great weight to matters which are very uncertain……. **By "very uncertain' I do not mean the same thing as "very improbable".."**(Davidson, 2011, p.38).

Davidson has deliberately misquoted Keynes by leaving out the reference to the A Treatise on Probability and chapter 6 of the TP.A reader who has followed Keynes in chapter 6 of the TP will immediately realize that Keynes has defined uncertainty to be an inverse function of weight .This definition has nothing to do with the homogeneity of data or the Keynes- Tinbergen debate over Tinbergen's misuse of the Normal probability distribution, which requires the axiom of additivity and the linearity property of the Normal distribution.

Davidson appears to be deliberately trying to mislead his readers .This is something that Davidson has done repeatedly over the last 33 years.

Let us now analyze Claim Five:

Davidson makes the following claim:

"Non -homogeneity is a sufficient condition for non ergodicity." (Davidson, 2012, p.62).

This is a false statement .See the contributions by Madsen and Isaacson,1973, Johnson and Isaacson,1988,Mukhamedov,2012, and Bisbas and Karanikas,1994 that establish that non homogeneity is not a sufficient condition for non ergodicity. Many other articles could have been cited.

Consider the erroneous discussion by Davidson concerning Taleb's analysis.

Davidson claims that

"Taleb's Black Swan concept attempts to explain market crashes as an event lying in the far off tail of an ergodic probability distribution. It should be noted that Knight's vision of uncertainty and Taleb's Black Swan concept are both based on the ergodic presumption for the economy. Taleb's Black Swan is an already predetermined outcome but the Black Swan event is so far out in the tail of the ergodic probability distribution that its occurrence is so rare that it is never likely to be observed– except in the long run when we will all be dead". (Davidson, 2012, p.62)

The distribution Davidson apparently does not know the name of is the Cauchy Distribution. The Cauchy distribution is not the finite mean-finite variance type of probability distribution that Davidson derived his very limited knowledge of probability from in his days in a biology department. It has an extremely large variance that supports a generalization to infinite variance. Its mean is also infinite. It has extreme kurtosis (peakedness) and fat tails with tail probability values that range from anywhere from 100 to 1000 times the size of the corresponding tail probabilities of a normal distribution, which is why Mandelbrot emphasized that comparisons of the Cauchy to the Normal distribution pointed to the far, far greater risk ("wild "risk) of the Cauchy versus the mild risk of the normal distribution emphasized by neoclassical economics.

Davidson's claim that "… its occurrence is so rare that it is never likely to be observed– except in the long run when we will all be dead" means that he doesn't understand what Taleb is talking about. Taleb's point ,discussed in more general terms by Keynes in chapter 26 of the TP, is that a small probability, which can't be calculated reliably or accurately using the Cauchy, can, when combined with an extremely large negative outcome, lead to catastrophic results .Of course, this distribution's

applicability has been verified well over 10,000 times by Mandelbrot, Taleb, and the hundreds of econophysics practitioners who have followed them .Nothing has been more empirically verified than the ergodicity of the financial markets. Davidson is simply ignorant of the fact that the weight of the evidence, based on assessments using the Cauchy Distribution, lead to probability assessments that are not reliable and hence have low weight. Given that uncertainty for Keynes is an inverse function of weight, Mandelbrot's and Taleb's results are not in conflict with Keynes's. In fact, they support them. Davidson's strange belief that ergodicity means computational accuracy and reliability of expected values, is completely wrong. See my 2015 SSRN paper listed in the Bibliography for a discussion of Davidson's confusions based on the fallacy of conditional a priorism that infect all of Davidson's work in this area.

Consider Claim number six:

"Non-stationarity is a sufficient but not a necessary condition for a nonergodic stochastic process. Accordingly, Keynes was implicitly arguing that economic processes over time occur in a nonergodic economic environment."(Davidson, 2009, p.332).

The first sentence is simply false. This is but one of many examples where Davidson simply keeps repeating his many errors over and over again.

.Keynes never argued that "…economic processes over time occur in a nonergodic economic environment". It is investment in fixed, long lived, durable capital goods, subject to the forces of technological innovation, advance, and obsolescence over time and expectations about future kinds of capital goods, that is the driving force in capitalistic economies. These kinds of investment are not stochastic processes because they occur in time as single events or infrequent events. There are no time series of such events. They occur in bunches.

Let us now consider Davidson's last claim. He repeats his canard that Keynes had no knowledge of the 1935 Moscow School of Probability. In fact M Keynes was an acknowledged, world renown, and internationally recognized expert in probability and statistics in the 1930's based on his A Treatise on Probability (1921). . Keynes had been selected by statistics journals to serve as a referee during the 1930's. It is, therefore, no surprise that he was selected as the referee by the League of Nations to review Jan Tinbergen's work on predicting business cycles that used an econometrics approach based on The Law of Large Numbers, the Central Limit Theorem, and the Gaussian (Normal) Distribution .The fundamental axiom used by Tinbergen was additivity.

Kolmogorov and the Moscow School of Probability's main innovation was to go from the axiom of additivity to the axiom of countable additivity. However, Keynes rejected additivity except in the special case that the weight of the evidence, w, which measured the relative completeness of the evidence ,defined on the closed unit interval [0,1],equaled 1 , approached 1,or approximated 1. Keynes also accepted goodness of fit tests, such as the Lexis –Q test, and exploratory data analysis as evidence that could be used to support using a particular probability distribution.

Both Ergodic and Nonergodic stochastic processes assume the axioms of measure theory as developed by Kolmogorov and others. One of these axioms was countable additivity (additivity). Keynes, in general, rejected the axiom of additivity (countable additivity).

Davidson is simply ignorant of what it was that the Moscow school of Probability was doing. Measure theoretical constructs, like ergodicity, were minor developments on the Strong Law of Large Numbers. Such a construct is valuable in the physical, life and engineering sciences, but is limited in the social and behavioral sciences, liberal arts, and especially in economics and business.

Section 5.) Non Additivity, the major axiom and non-linearity, the minor axiom, were J M Keynes's two fundamental axioms

Davidson has claimed for well over 30 years that three of Keynes's "conclusions" ,"non ergodicity" , the non-neutrality of money ,and non-gross substitutability were "axioms". Similarly, Davidson has claimed that the three neoclasssical conclusions, ergodicity , the neutrality of money, and gross substitutability were also "axioms"'. Davidson has confused axioms with the conclusions that follow from the axioms. His work is based on confusions. No neoclassical economist has ever stated that ergodicity is a fundamental axiom .Keynes never stated that non ergodicity is one of his fundamental axioms. Keynes did state that he rejected additivity. Interval valued probability and Keynes's c coefficient analysis follow directly from Keynes's acceptance of non additivity.

Keynes's fundamental axiom is thus non additivity .Non linearity would be Keynes's next axiom. A simplified version of these axioms would state that there is incomplete and imperfect information, data, and knowledge in the long run for all decision makers.

By contrast, the basic and fundamental neoclassical axioms are additivity and linearity. They can be reformulated and described in simpler terms as the assumption of complete and perfect information, data, and knowledge in the long run for all decision makers.

Keynes 's basic approach to probability, using either his interval valued probability approach ,which can be expressed mathematically with the aid of inequality constraints ,or his conventional coefficient of weight and risk ,c, both involve non additivity and non-linearity , in general ,with additivity and linearity as limiting conditions.

Non-additivity is a necessary condition for uncertainty, in general. It is non additivity that leads to the rejection of the neutrality of money and gross substitutability conclusions of neoclassical theory, which require addiitvity. Additivity, where the weight of the evidence, w, is defined on the unit interval between 0 and 1, requires a w=1. Non additivity results if w<1 and leads directly to the conclusion that there is uncertainty. If w=1, then one is led to the conclusion that all decisions are risky .No money or money substitutes need be held in this case. An identical form of analysis hold with respect to the conclusion concerning gross substitutability.

Section 6.) Conclusions

The fundamental axioms of Keynes's non Euclidean economics, non additivity and non-linearity, are more general than the neoclassical, Euclidean axioms of additivity and linearity because they include the neoclassical axioms as special cases that occur when w,the weight of the evidence,approaches,equals,or approximates w=1. Thus, the Ramsey –de Finetti –Savage-Friedman marriage of subjective probability to the Von Neumann –Morgenstern expected utility model results in a special theory, Subjective Expected Utility (SEU), which is a special case of Keynes's general decision theory.SEU theory requires linearity and additivity.

P. Davidson's work since 1983 on the axiomatic foundations of Keynes's work is confused and error filled. It directly contradicts Keynes 's approach in both the TP and GT .It confuses axioms with conclusions and mixes up Keynes' s logical theory of probability with Davidson's preferred theory of probability, the relative frequency theory of probability of Kolmogorov .Kolmogorov's limiting frequency version of probability ,upon which Kolmogorov built his ergodicity approach ,is then erroneously used to examine Keynes's work on uncertainty.

References

Anily, S. and Federgruen, A. (1987). "Ergodicity in Parametric Nonstationary Markov Chains: An Application to Simulated Annealing Methods." *Operations Research*, 1987, 35, No.6, (November-December) 867-874.

Boole, George. 1854. *An Investigation of the Laws of Thought on Which are founded the Mathematical Theories of Logic and Probability*. New York: Dover Publications, 1954

Brady, Michael Emmett. (2011).A Comparison-Contrast of Adam Smith, JM Keynes and Jeremy Bentham on Probability, Risk, Uncertainty, Optimism-Pessimism and Decision Making with Applications Concerning Banking, Insurance and Speculation.
International Journal of Applied Economics and Econometrics, Volume 19, No.4, October -December ,pp. 86-111.

Brady, Michael Emmett. (2015).Adam Smith's Theory of Probability and the Roles of Risk and Uncertainty in Economic Decision Making.
International Journal of Applied Economics and Econometrics, Vol. 23, no.1,2015.

Brady, Michael E.(2015).Paul Davidson, the Post Keynesians, and the Inductive Fallacy of Conditional A priorism (Long Runism)-Why the Ergodic-Non Ergodic distinction is meaningless and erroneous for ALL Decision Makers. Available at SSRN.

Brady, Michael E. 1993. "J.M. Keynes' theoretical approach to decision making under condition of risk and uncertainty." *The British Journal for the Philosophy of Science* 44: 357-76.

Brady, Michael E. 2002. "J.M. Keynes's Theory and Preference Reversals." *International Journal of Applied Economics and Econometrics* 1: 301-08.

Brady, Michael E. 2004A. *J.M. Keynes' Theory of Decision Making, Induction, and Analogy. The Role of Interval Valued Probability in His Approach*. Bloomington: Xlibris Corporation.

Brady, Michael E. 2004B. *Essays on John Maynard Keynes and ….* Bloomington: Xlibris Corporation.

Brady, Michael E., Arthmar, Rogério. 2012. "Keynes, Boole and the interval approach to probability." *History of Economic Ideas*, XX (3): 65-84.

Brady, Michael Emmett. (2012). Keynes, Mathematics and Probability: A Reappraisal. International Journal of Applied Economics and Econometrics, Volume 20, no. 2, April-June 2012, pp.324-357Broad, C.D. 1922. "A Treatise on Probability by J.M. Keynes." *Mind*, 31: 72–85.

Edgeworth, Francis Y. 1922. "Review of A Treatise on Probability." *Journal of the Royal Statistical Society* 85: 107-13.

Edgeworth, Francis Y. 1922. "The Philosophy of Chance." *Mind* 31: 157-285.

Davidson, Paul. (1982-83).Rational Expectations: A fallacious foundation for studying crucial decision-making processes, Journal of Post Keynesian Economics, Vol. 5, No.2, pp. 182-198.

Davidson, Paul. (n.d.).SAMUELSON AND THE KEYNES/POST KEYNESIAN REVOLUTION: THE EVIDENCE SHOWING WHO KILLED COCK ROBIN, p.18.

Davidson Paul. (2005). STRONG UNCERTAINTY AND HOW TO COPE WITH IT TO IMPROVE ACTION AND CAPACITY. EAEPE Conference. Bremen, Germany. November 11, 2005.

Davidson, Paul. (2008). Post World War II politics and Keynes's aborted revolutionary economic theory.,Economia e Sociedade, Campinas, v. 17, Número especial, p. 549-568, Dec. 2008 .

Davidson, Paul. (2009).Can future systemic financial risks be quantified? Ergodic vs. Nonergodic stochastic processes**.** Brazilian Journal of Political Economy, vol. 29, nº 4 (116), pp. 324-340, October-December/2009

Davidson, Paul. (2012). "Is economics a science**?** Should economics be rigorous? ", *real-world economics review*, issue no. 59, 12 March 2012, pp. 58-66,**.**

Franklin, Johanna and Towsner, Henry. (2013).Randomness and Non-Ergodic Systems. NSF grant DMS-1340666.

Johnson, Jean and Isaacson, Dean. (1988).Conditions for Strong Ergodicity Using Intensity matrices. Journal of Applied Probability, Vol. 25, No.1 (March 1988), pp.34-42.

Keynes, John Maynard. (1921). a Treatise on Probability. England: Macmillan.

Keynes, J. M. (1921). *A Treatise on Probability. The Collected Writings of John Maynard Keynes* (*CWJMK*) vol. VIII. London: Macmillan, 1973

Keynes, John Maynard. (1936).The General Theory of Employment, Interest and Money. New York: Harcourt, Brace and World.

Madsen, R.W. and Isaacson, D.L. (1973) .Strongly Ergodic Behavior for Non Stationary Markov Processes." *Annals of Probability*, 1973, *1*(2), 329-335.

Mukhamedov, F. (2012).On L_1 –Weak Ergodicity of Non homogeneous Discrete Processes and Its Applications.

Shinkai, S. and Aizawa, Y. "The Lempel-Ziv Complexity of Non-Stationary Chaos in Infinite Ergodic Cases." *Progress of Theoretical Physics*, 2006, *116*(3), 503-515.

Smith, Adam. (1776).The Wealth of Nations. Modern (Cannan) Library Edition with the foreword by Max Lerner. New York, New York .

Smith, Adam. (1759). the Theory of Moral Sentiments. Sixth edition, 1790. Gutenberg Publishers (2011).