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USING THE COMPREHENSIVE (G'/G)-EXPANSION METHOD TO INQUIRE ABOUT THE NEW PRECISE SOLUTIONS OF THE (2+1)-DIMENSIONAL DISPERSIVE LONG-WAVE EQUATIONS

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ABSTRACT

With the help of computer algebra system mathematical, this article uses the extended (G'/G)-expansion method to obtain travelling wave solutions to (2+1) -dimensional dispersive long-wave equations, which are represented by the hyperbolic functions and trigonometric functions with arbitrary parameter.

Keywords: *Long Wave Equations, Dimensional Dispersive, Expansion Method.*

1. INTRODUCTION

The (2+1)-dimensional dispersive long wave equations[1~3]

$$\begin{cases} u_{yt} + v_{xx} + \frac{1}{2}(u^2)_{xy} = 0 \\ v_t + (uv + u + u_{xy})_x = 0 \end{cases} \tag{1}$$

are important equations in describing the motion of water waves traveling through ideal deep, long and narrow waterway. For a long time, the solutions of them have drawn many scholars' attention. In recent years, with the development of computer science, people have built up a lot of effective method for solving nonlinear evolution equations , such as Bäcklund transform method, Darboux transform method, Homogeneous balance method[4], similarity reduction method, auxiliary equation method, hyperbolic function method, inverse scattering transform method, Jacobi elliptic function expansion method, F-expansion method[5~8], Hirota bilinear method and so on. At present, with the help of symbolic computation system like Mathematica, Solitary wave solutions , Soliton solutions, Periodic solutions and other type of exact solutions to the (2 +1)-dimensional dispersive long-wave equations have been worked out one after another. Recently, (G'/G)-expansion method[9~13] is proposed to provide an effective method for solving nonlinear evolution equations. Now, This method has been applying to solving a number of nonlinear evolution equations, which is turned out to be effective.

This paper will take an extension to (G'/G)-expansion method, which is, for more specific, the positive power is extended to the positive and negative power. By using this method, we succeed in working out the traveling wave solutions to the (2 +1)-dimensional dispersive long wave equations in the form of hyperbolic functions.

2. Introduction of the extended (G'/G)- expansion method

Given nonlinear PDE, with two independent variables x and t:

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0 \tag{2}$$

Where P is the polynomial of u, which contains several higher-order derivatives and nonlinear terms. The detailed steps in solving(2) using extended (G'/G) -expansion method are as follows[2-4]:

(Ⅰ)Wave reduction of Eq.(2). Let

$$u(x, y, t) = u(\xi), \xi = x + \lambda y + \mu t \tag{3}$$

Where λ and μ are undetermined constants, substituting Eq.(3) into (2) , can be written as an ODE about $\mu(\xi)$:

$$Q(u, u', u'', \ldots) = 0 \tag{4}$$

(Ⅱ)Set $\mu(\xi)$ as a finite series

$$u(\xi) = \sum_{i=-n}^{n} a_i \left(\frac{G'}{G}\right)^i \tag{5}$$

Where $a_i (i = -n, \ldots, -1, 0, 1, \ldots, n)$ are constants to be determined, and $G(\xi)$ meets the following second-order linear ordinary differential equations

$$G''(\xi) + aG'(\xi) + bG(\xi) = 0 \tag{6}$$

Positive integer n can be determined by homogeneous balance principle.

(Ⅲ)Substitute Eq.(5) into (4),transform the left half of (4) to polynomial of (G'/G) [9] and let the polynomial coefficients be zero, then algebraic equations with $a_i (i = -n, \ldots, -1, 0, 1, \ldots, n), \lambda, \mu$ can be obtained.

(Ⅳ) Solve the above algebraic equations using mathematica, substitute the results of $a_i (i = -n, \ldots, -1, 0, 1, \ldots, n), \lambda, \mu$ we got into the Eq.(4), then we obtain the exact traveling wave solutions of the nonlinear equations (1).

3 Calculate the exact solution of the (2 +1)-dimensional Dispersive Long Wave Equation

(2 +1)-dimensional dispersive long wave equation (1) is given in the form of two separate partial differential equations, as follows:

$$v_t + (uv + u + u_{xy})_x = 0. \tag{7}$$

$$u_{yt} + v_{xx} + \frac{1}{2}(u^2)_{xy} = 0 \tag{8}$$

Assuming traveling wave solutions as follows exists:

$$u(x, y, t) = u(\xi), v(x, y, t) = v(\xi), \xi = x + \lambda y - \mu t \tag{9}$$

Then Eqs. (7) and (8)can be transformed into ordinary differential equations about $u = u(\xi), v = v(\xi)$:

$$-\lambda\mu u''(\xi) + v''(\xi) + \frac{\lambda}{2}(u^2(\xi))'' = 0, \tag{10}$$

$$-\mu v'(\xi)+(u(\xi)v(\xi)+\lambda u''(\xi))'=0. \qquad (11)$$

We integrate ξ on Eqs.(10) and (11) respectively for one time, then we will get

$$-\lambda\mu u'(\xi)+v'(\xi)+\lambda u(\xi)u'(\xi)+c_1=0, \qquad (12)$$

$$-\mu v(\xi)+u(\xi)v(\xi)+\lambda u''(\xi)+c_2=0. \qquad (13)$$

Where c_1 and c_2 are undetermined constants.

Basing on the homogeneous balance principle, balancing the

v', uu' of (12) and u'', uv of (13) ,we required that $m_1=1, m_2=2$. We can suppose that the solutions of Eqs.(12) and (13) are as follows:

$$u(\xi)=-a_{-1}\left(\frac{G'}{G}\right)^{-1}+a_0+a_1\left(\frac{G'}{G}\right), a_{-1}, a_1\neq 0, \qquad (14)$$

$$v(\xi)=b_{-2}\left(\frac{G'}{G}\right)^{-2}+b_{-1}\left(\frac{G'}{G}\right)^{-1}+b_0+b_2\left(\frac{G'}{G}\right)^{2}+b_1\left(\frac{G'}{G}\right)+b_0, b_2\neq 0. \qquad (15)$$

Where $G=G(\xi)$ satisfies the second order linear ordinary differential equations (LODE) equation

$$G''+\beta G'+\gamma G=0 \qquad (16)$$

Where $a_{-1}, a_1, a_0, b_{-2}, b_{-1}, b_2, b_1, b_0, \beta$ and γ are undetermined coefficients. According to the expression (14) to (16) ,we can obtain the following equations;

$$u'(\xi)=a_{-1}\gamma(\frac{G'}{G})^{-2}+a_{-1}\beta(\frac{G'}{G})^{-1}-a_1\beta(\frac{G'}{G})-a_1(\frac{G'}{G})^{2}+a_{-1}-a_1\gamma \qquad (17)$$

$$u''(\xi)=2a_{-1}\gamma^2(\frac{G'}{G})^{-3}+3a_{-1}\beta\gamma(\frac{G'}{G})^{-2}+(a_{-1}\beta^2+2a_{-1}\gamma)(\frac{G'}{G})^{-1}-(2a_1\gamma+a_1\beta^2)(\frac{G'}{G})$$
$$+3a_1\beta(\frac{G'}{G})^{2}+2a_1(\frac{G'}{G})^{3}+a_{-1}\beta+a_1\beta\gamma \qquad (18)$$

$$v'(\xi)=2b_{-2}\gamma(\frac{G'}{G})^{-3}+(2b_{-2}\beta+b_{-1}\gamma)(\frac{G'}{G})^{-2}+(2b_{-2}+b_{-1}\beta)(\frac{G'}{G})^{-1}$$
$$-(b_1\beta+2b_2\gamma)(\frac{G'}{G})-(b_1+2b_2\beta)(\frac{G'}{G})^{2}-2b_2(\frac{G'}{G})^{3}+b_{-1}-b_1\gamma \qquad (19)$$

substitute Eqs.(18) and (19) equation into Eqs.(12), (13), integrate homogeneous items with (G'/G),then Eqs.(12) and (13) can be transformed to the polynomial of(G'/G),therefore a series of algebraic equations about $a_{-1}, a_1, a_0, b_{-2}, b_{-1}, b_2, b_1, b_0, \mu, \beta, \gamma$ and c_1, c_2 can be gotten:

$$\begin{cases} 2b_{-2}\gamma+\gamma a_{-1}{}^2=0, \\ -\lambda\mu a_{-1}\gamma+2b_{-2}\beta+b_{-1}\gamma+\lambda\beta a_{-1}{}^2+\lambda\gamma a_0 a_{-1}=0, \\ -\lambda\beta\mu a_{-1}+2b_{-2}+\beta b_{-1}+\lambda a_{-1}{}^2-\lambda\gamma a_{-1}a_1+\lambda\beta a_0 a_{-1}+\lambda\gamma a_{-1}a_1=0, \\ \lambda\mu\beta a_1-\beta b_1-2\gamma b_2-\lambda\beta a_0 a_1-\lambda\gamma a_1{}^2=0, \\ \lambda\mu a_1-\lambda a_0 a_1-\lambda\beta a_1{}^2-b_1-2b_2\beta=0, \\ -2b_2-\lambda a_1{}^2=0, \\ -\lambda\mu a_{-1}+\lambda\mu\gamma a_1+b_{-1}-b_1\gamma+\lambda a_0 a_1-\lambda\gamma a_0 a_1+c_1=0 \end{cases} \tag{20}$$

and

$$\begin{cases} a_{-1}b_{-2}+2a_{-1}\gamma^{2\lambda}=0, \\ a_{-1}b_{-1}-\mu b_{-2}+a_0 b_{-2}+3\gamma a_{-1}\beta\lambda=0, \\ -\mu b_{-1}+a_{-1}b_0+a_0 b_{-1}+a_1 b_{-2}+a_{-1}\lambda\beta^2=0, \\ -\mu b_1+b_2 a_{-1}+a_0 b_1+a_1 b_0+2\gamma\lambda a_1+a_1\lambda\beta^2=0, \\ -\mu b_2+a_0 b_2+a_1 b_1+3a_1\beta\lambda=0, \\ a_1 b_2+2a_1\lambda=0, \\ -\mu b_0+a_{-1}b_1+a_0 b_0+a_1 b_{-1}+\gamma a_1\lambda\beta+c_2+\beta\lambda a_{-1}=0 \end{cases} \tag{21}$$

Solving above algebra equation by using mathematica, we obtain four groups of results as follows:

$$\begin{aligned} &a_{-1}=0, a_0=-\beta+\mu, a_1=-2, b_{-2}=0, b_{-1}=0, \\ &b_0=-2\gamma\lambda, b_1=-2\beta\lambda, b_2=-2\lambda, c_1=0, c_2=0; \end{aligned} \tag{22}$$

$$\begin{aligned} &a_{-1}=-2\gamma, a_0=-\beta+\mu, a_1=0, b_{-2}=-2\gamma^2\lambda, b_{-1}=-2\gamma\beta\lambda, \\ &b_0=0, b_1=0, b_2=0, c_1=0, c_2=2\gamma\beta\lambda; \end{aligned} \tag{23}$$

$$\begin{aligned} &a_{-1}=-2\gamma, a_0=\beta+\mu, a_1=0, b_{-2}=-2\gamma^2\lambda, b_{-1}=-2\gamma\beta\lambda, \\ &b_0=0, b_1=0, b_2=0, c_1=0, c_2=-2\gamma\beta\lambda; \end{aligned} \tag{24}$$

$$\begin{aligned} &a_{-1}=0, a_0=\beta+\mu, a_1=2, b_{-2}=0, b_{-1}=0, \\ &b_0=-2\gamma\lambda, b_1=-2\beta\lambda, b_2=-2\lambda, c_1=0, c_2=0; \end{aligned} \tag{25}$$

Where β , λ and μ are constant.

Substituting the results of Eq.(22) into Eqs.(14) and (15), we can obtain the solutions of Eqs. (12) and (13) .

$$\begin{cases} u_1(\xi)=\pm 2\left(\dfrac{G'}{G}\right)+\mu\pm\beta, \\ v_1(\xi)=-2\lambda\left(\dfrac{G'}{G}\right)^2-2\lambda\beta\left(\dfrac{G'}{G}\right)-2\lambda\gamma; \end{cases} \tag{26}$$

$$\begin{cases} u_2(\xi)=\pm 2\gamma(\dfrac{G'}{G})^{-1}+\mu\pm\beta, \\ v_2(\xi)=-2\gamma^2\lambda\left(\dfrac{G'}{G}\right)^{-2}-2\gamma\beta\lambda\left(\dfrac{G'}{G}\right)^{-1}. \end{cases} \tag{27}$$

$\xi = x + \lambda y - \mu t.$

The general solutions of Eq. (16) follows, part of which are also given in Ref.[2] and [3]:

$$(G'/G) = \begin{cases} \frac{\sqrt{\beta^2-4\gamma}}{2}\left(\frac{A_1 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}{A_2 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_1 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}\right) - \frac{\beta}{2}, \beta^2 - 4\lambda > 0, \\ \frac{\sqrt{4\lambda-\beta^2}}{2}\left(\frac{-A_1 \sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi + A_2 \cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}{A_2 \sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi + A_1 \cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}\right) - \frac{\beta}{2}, \beta^2 - 4\lambda < 0, \\ \frac{A_2}{A_1 + A_2\xi} - \frac{\beta}{2}, \beta^2 - 4\mu = 0. \end{cases}$$

(28)

Substituting Eq.(28) into Eqs.(26) and (27), we finally obtain the exact travelling wave solutions of the (2+l)-dimensional long-wave equation (1):

1) When $\beta^2 - 4\gamma > 0$, we have hyperbolic function solution:

$$u_{1,1,2}(\xi) = \pm\sqrt{\beta^2-4\gamma}\left(\frac{A_1 \sinh(\frac{1}{2}\sqrt{\beta^2-4\gamma\xi}) + A_2 \cosh(\frac{1}{2}\sqrt{\beta^2-4\gamma\xi})}{A_2 \sinh(\frac{1}{2}\sqrt{\beta^2-4\gamma\xi}) + A_1 \cosh(\frac{1}{2}\sqrt{\beta^2-4\gamma\xi})}\right) + \mu,$$

$$u_{2,1,2}(\xi) = \pm 2\gamma\left(\frac{\sqrt{\beta^2-4\gamma}}{2}\left(\frac{A_1 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}{A_1 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}\right) - \frac{\beta}{2}\right)^{-1} + \mu \pm \beta,$$

$$v_{1,1,2}(\xi) = -2\lambda\left(\frac{\sqrt{\beta^2-4\gamma}}{2}\left(\frac{A_1 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}{A_1 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}\right) - \frac{\beta}{2}\right)^2 +$$

$$-2\lambda\beta\left(\frac{\sqrt{\beta^2-4\gamma}}{2}\left(\frac{A_1 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}{A_1 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}\right) - \frac{\beta}{2}\right) - 2\lambda\gamma,$$

$$v_{2,1,2} = -2\gamma^2\lambda\left(\frac{\sqrt{\beta^2-4\gamma}}{2}\left(\frac{A_1 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}{A_1 \cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi + A_2 \sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}\right) - \frac{\beta}{2}\right)^{-2} -$$

$$2\gamma\beta\lambda\left(\frac{\sqrt{\beta^2-4\gamma}}{2}\left(\frac{A_1\sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi+A_2\cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}{A_1\cosh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi+A_2\sinh\left(\frac{1}{2}\sqrt{\beta^2-4\gamma}\right)\xi}\right)-\frac{\beta}{2}\right)^{-1}.$$

where $\xi = x+\lambda y-\mu t$, A_1 and A_2 are arbitrary constant.

2) When $\beta^2-4\gamma<0$, we have trigonometric function solution:

$$u_{1,3,4}(\xi)=\pm\sqrt{4\lambda-\beta^2}\left(\frac{-A_1\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}{A_1\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}\right)+\mu,$$

$$u_{2,3,4}(\xi)=\pm2\gamma\left(\frac{\sqrt{4\lambda-\beta^2}}{2}\left(\frac{-A_1\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}{A_1\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}\right)-\frac{\beta}{2}\right)^{-1}+\mu\pm\beta,$$

$$v_{1,3,4}(\xi)=-2\lambda\left(\frac{\sqrt{4\lambda-\beta^2}}{2}\left(\frac{-A_1\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}{A_1\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}\right)-\frac{\beta}{2}\right)^{2}-$$

$$\lambda\beta\sqrt{4\lambda-\beta^2}\left(\frac{-A_1\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}{A_1\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}\right)-\lambda\beta^2-2\lambda\gamma,$$

$$v_{2,3,4}(\xi)=-2\gamma^2\lambda\left(\frac{\sqrt{4\lambda-\beta^2}}{2}\left(\frac{-A_1\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}{A_1\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}\right)-\frac{\beta}{2}\right)^{-2}-$$

$$2\gamma\beta\lambda\left(\frac{\sqrt{4\lambda-\beta^2}}{2}\left(\frac{-A_1\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}{A_1\cos\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi+A_2\sin\left(\frac{1}{2}\sqrt{4\lambda-\beta^2}\right)\xi}\right)-\frac{\beta}{2}\right)^{-1}$$

where $\xi = x+\lambda y-\mu t$, A_1 and A_2 are arbitrary constant.

3) When $\beta^2-4\gamma=0$, we have rational solution:

$$u_{1,5,6}(\xi)=\frac{\pm2A_2}{A_1+A_2\xi}+\mu,$$

$$u_{2,5,6}(\xi)=\pm 2\gamma\left(\frac{A_2}{A_1+A_2\xi}-\frac{\beta}{2}\right)^{-1}+\mu\pm\beta,$$

$$v_{1,5,6}(\xi)=-2\lambda\left(\frac{A_2}{A_1+A_2\xi}-\frac{\beta}{2}\right)^2-2\lambda\beta\frac{A_2}{A_1+A_2\xi}+\lambda\beta^2-2\lambda\gamma,$$

$$v_{1,5,6}(\xi)=-2\gamma^2\lambda\left(\frac{A_2}{A_1+A_2\xi}-\frac{\beta}{2}\right)^{-2}-2\gamma\beta\lambda\left(\frac{A_2}{A_1+A_2\xi}-\frac{\beta}{2}\right)^{-1}.$$

where $\xi=x+\lambda y-\mu t$, A_1 and A_2 are arbitrary constants.

4. Conclusions

In this paper, by using the homogenous balance and the (G'/G)-expansion method ,we work out the new exact solutions of the (2+l)-dimensional long-wave equations, which are in the form of hyperbolic functions and trigonometric functions containing arbitrary parameters. We get great benefit from the advanced mathematica computer algebra system. As the Eq.(16) has many other solutions of functional forms, we can further get more solutions that have different structures.

Additionally, by solving the (2+l)-dimensional long-wave equations, we find out that the extended (G'/G)-expansion method is much succinct and effective, it can widely be applied to other nonlinear evolution equations.

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